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Baryon spectra and AdS/CFT correspondence

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Abstract: We provide a detailed map between wrapped D3-branes in Anti-de Sitter (AdS) backgrounds and dibaryon operators in the corresponding conformal field theory (CFT). The effective five dimensional action governing the dynamics of AdS space contains a $U(1)_R$ gauge field that mediates interactions between objects possessing R-charge. We show that the $U(1)_R$ charge of these wrapped D3-branes as measured by the gauge field matches the R-charge of the dibaryons expected from field theory considerations. We are able, through a careful probe brane calculation in an $AdS_5 \times T^{1,1}$ background, to understand the exact relation between the mass of the wrapped D3-brane and the dimension of the corresponding dibaryon. We also make some steps toward matching the counting of dibaryon operators in the CFT with the ground states of a supersymmetric quantum mechanical system whose target space is the moduli space of D-branes. Finally, we discuss BPS excitations of the D3-brane and compare them with higher dimension operators in the CFT.

Keywords: D-branes, Brane Dynamics in Gauge Theories, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

According to the AdS/CFT conjecture [1, 2, 3], the chiral operators of the $\mathcal{N} = 4$ supersymmetric SU(N) gauge theory are in one-to-one correspondence with the modes of type-IIB supergravity on $AdS_5 \times S^5$. On the other hand, the massive string modes correspond to operators in long multiplets whose dimensions diverge for large 't Hooft coupling as $(g^2_{\mathrm{YM}} N)^{1/4}$. Thus, in the limit $g^2_{\mathrm{YM}} N \to \infty$ the stringy nature of the dual theory is obscured by the decoupling of the non-chiral operators, which constitute the large majority of possible gauge invariant operators.

It is important to keep in mind, however, that the AdS/CFT correspondence relates large N gauge theory to string theory, not merely supergravity. If we depart from the limit of infinite 't Hooft coupling, then all the non-chiral operators no longer decouple, so that the spectrum of the gauge theory becomes much more complicated and presumably related to type-IIB string theory on $AdS_5 \times S^5$ (see, for example, [4]). However, even for very large 't Hooft coupling, it is possible to demonstrate the stringy nature of the dual theory quite explicitly. One early clue was provided by Witten [5] in the context of $\mathcal{N} = 4$ supersymmetric SO($2N$) gauge theory which is dual to type-IIB strings on $AdS_5 \times RP_5$. He noted that the gauge theory possesses chiral operators of dimension N, the Pfaffians, whose dual is provided by a D3-brane wrapping a 3-cycle of the RP_5. This explicitly shows that the dual theory cannot be simply supergravity: it must contain D-branes. Since the third homology class $H_3(RP_5) = \mathbb{Z}_2$, a D3-brane wrapped twice can decay into ordinary supergravity modes. A corresponding gauge theory statement is that a product of two Pfaffian operators can be expressed in terms of the usual single-trace operators.

It is further possible to find type-IIB backgrounds of the form $AdS_5 \times \mathbf{X}_5$ such that $H_3(\mathbf{X}_5) = \mathbb{Z}$. In this case, a D3-brane can wrap a 3-cycle any number of times so that the dual gauge theory operators carry a quantized baryon number. A simple example of this kind is provided by the space $\mathbf{X}_5 = T^{1,1} = (\mathrm{SU}(2) \times \mathrm{SU}(2))/\,\mathrm{U}(1)$ whose symmetries are $\mathrm{U}(1)_R \times \mathrm{SU}(2) \times \mathrm{SU}(2)$. This space may be thought of as a U(1) bundle over $S^2 \times S^2$, and the explicit Einstein metric on $T^{1,1}$ is

$$ds_5^2 = \frac{1}{9}\left(d\psi + \sum_{i=1}^{2} \cos\theta_i d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right). \tag{1.1}$$

The $\mathcal{N}=1$ superconformal gauge theory dual to type-IIB strings on $AdS_5 \times T^{1,1}$ was constructed in [6]: it is $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory coupled to two chiral superfields, A_i, in the $(\mathbf{N}, \overline{\mathbf{N}})$ representation and two chiral superfields, B_j, in the $(\overline{\mathbf{N}}, \mathbf{N})$ representation [6]. The A's transform as a doublet under one of the global SU(2)'s while the B's transform as a doublet under the other SU(2).

Cancellation of the anomaly in the U(1) R-symmetry requires that the A's and the B's each have R-charge 1/2. For consistency of the duality it is necessary that we add an exactly marginal superpotential which preserves the $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)_R$ symmetry of the theory:

$$W = \epsilon^{ij}\epsilon^{kl}\,\mathrm{tr}\,A_iB_kA_jB_l\,. \tag{1.2}$$

In [7] it was proposed that the wrapped D3-branes correspond to baryon-like operators A^N and B^N where the indices of both SU(N) groups are fully antisymmetrized (their more detailed form is exhibited in section 3). For large N the dimensions of such operators calculated from the mass of wrapped D3-branes were found to be $3N/4$ [7]. This is in complete agreement with the fact that the dimension of the chiral superfields at the fixed point is 3/4. At the quantum level, the collective coordinate for the wrapped D3-brane has to be quantized to identify its $\mathrm{SU}(2) \times \mathrm{SU}(2)$ quantum numbers. This quantization was sketched in [7]. Here we do a more careful job and show that the dimension of dibaryons is $3N/4$ not only in the large N limit but exactly. In a related check on the identification of the dibaryons with wrapped D3-branes, we calculate the $\mathrm{U}(1)_R$ charge of the wrapped D3-branes and show that it is $N/2$, in agreement with the gauge theory.

The structure of the paper is as follows. In section 2 we calculate the $\mathrm{U}(1)_R$ charge of a D3-brane wrapping a 3-cycle inside the Einstein space $\mathbf{X}_5$. Our results apply to all $\mathbf{X}_5$ which are U(1) bundles over 4-d Kähler-Einstein spaces. We also provide an analogous calculation for M5-branes wrapping a 5-cycle inside an Einstein space $\mathbf{X}_7$ which is a U(1) bundle over a 6-d Kähler-Einstein space. In all cases, we show that the R-charge is proportional to the dimension Δ of the dual CFT operator, with the correct constant of proportionality. We further argue that Δ is measured by the volume of the cycle wrapped by the brane. In section 3 we make some remarks on the collective coordinate quantization of the wrapped D3-branes. In particular, we focus on the $AdS_5 \times T^{1,1}$ example and show that the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ quantum numbers of the dibaryon operators indeed follow from this quantization. In section 4 we study BPS excitations of the wrapped D3-branes using the DBI action. We calculate their energies and $\mathrm{U}(1)_R$ charges and propose dual chiral operators carrying the same quantum numbers.

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2. The R-symmetry gauge field

We would like to show how the $\mathrm{U}(1)_R$ gauge field emerges from a Kaluza-Klein reduction of the full ten or eleven dimensional supergravity action.[1] In ten dimensions, we begin with a stack of D3-branes placed at the tip of a non-compact Calabi-Yau cone $\mathbf{Y}_3$ of complex dimension three. In eleven dimensions, we consider a stack of M2-branes at the tip of a similar cone $\mathbf{Y}_4$ of complex dimension 4. We may write the metric on $\mathbf{Y}_n$ as

$$ds^2_{\mathbf{Y}_n} = dr^2 + r^2 ds^2_{\mathbf{X}_{2n-1}} , \tag{2.1}$$

where $\mathbf{X}_{2n-1}$ is Einstein, i.e. $R_{ab} = 2(n-1)g_{ab}$.

In the near-throat limit [1], the backreaction of the branes causes the full space to separate into a product manifold. In the D3-brane case, we have $AdS_5 \times \mathbf{X}_5$ and a five form flux $dC_4 = F_5$ produced by the stack of D3-branes. See also [9] for details. For the M2-branes, the full space becomes $AdS_4 \times \mathbf{X}_7$, and the M2-brane flux is carried by $dC_3 = F_4$.

When $\mathbf{X}_5$ is S^5 or the coset manifold $T^{1,1}$, the authors of [10, 11] demonstrate that perturbations in the metric $h_{\mu i}$ combine with perturbations in the $C_{\mu ijk}$ potential to produce a massless vector field. (The $i, j, \ldots$ index $\mathbf{X}_m$ while $\mu, \nu, \ldots$ index AdS space.) For $\mathbf{X}_7$, more general results exist concerning the existence of this massless vector field [12]. In particular, whenever $\mathbf{X}_7$ is a coset space and there exists a Killing vector, harmonic analysis demonstrates the existence of a massless vector field mixing perturbations of $h_{\mu i}$ with perturbations to $C_{\mu ij}$.

In what follows, we consider a more general class of $\mathbf{X}_n$, in particular quasi-regular Einstein-Sasaki manifolds, and demonstrate the existence of a massless U(1) vector field. We argue that this massless vector field mediates the interaction between objects with R-charge. Although the generalization is interesting in its own right, the main interest is the demonstration that wrapped D3-branes and M2-branes in these geometries always carry an R-charge consistent with their identification as dibaryons in the corresponding conformal field theory.

To be more specific, a quasi-regular Einstein-Sasaki manifold is essentially a U(1) fibration of a Kähler-Einstein manifold or orbifold. The BPS dibaryons wrap the base of the Sasaki-Einstein manifold in a holomorphic manner, and they wrap the S^1 fiber completely. Mikhailov has considered similar holomorphically wrapped D-branes [13]. One might naively say that the R-charge of a particle is its angular momentum along the S^1 direction. Thus, a brane that wraps the fiber carries no net angular momentum. However, one must be careful; these branes can be supersymmetric, and it is necessary to understand what the precise notion of R-charge is in the supergravity. The following calculation is not that sensitive to the dimensionality, but for clarity, we will consider the AdS_4 and AdS_5 cases separately.

[1]This gauge field is also discussed in [8] where the presence of fractional branes causes the field to acquire a mass.

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2.1 The R-charge for $AdS_5 \times \mathbf{X}_5$

In general, the ten dimensional metric is

$$ds^2 = \frac{r^2}{L^2}\eta_{\alpha\beta}dx^\alpha dx^\beta + L^2\frac{dr^2}{r^2} + L^2 ds^2_{\mathbf{X}_5}, \tag{2.2}$$

where $\eta_{\alpha\beta} = (-+++)$ is a Minkowski tensor.

A solution to the SUGRA equations of motion can be obtained by threading the $\mathbf{X}_5$ space with N units of F_5 form flux. In particular

$$F_5 = \mathcal{F} + \star\mathcal{F}\,; \qquad \mathcal{F} = 4L^4\mathrm{vol}(\mathbf{X}_5) \tag{2.3}$$

where $L^4 \sim g_s N$.

We now further specialize to the case where $\mathbf{X}_5$ is a quasi-regular Einstein-Sasaki space. In simpler language, $\mathbf{X}_5$ is a U(1) bundle over a two complex dimensional Kähler-Einstein manifold (or orbifold) V:

$$ds^2_{\mathbf{X}_5} = \left(\frac{q}{3}\right)^2\left(d\psi + \frac{3}{q}\sigma\right)^2 + h_{\alpha\bar{\beta}}dz^\alpha d\bar{z}^\beta\,. \tag{2.4}$$

Define the Kähler form on V to be $\omega = ih_{\alpha\bar{\beta}}dz^\alpha \wedge d\bar{z}^\beta$. Then, $d\sigma = 2\omega$. The number q is defined such that $n\,d\sigma = 2\pi q c_1$, where c_1 is the first Chern class of the U(1) bundle and $n = 3$. With these definitions, $0 \le \psi < 2\pi$ [14]. Also, in this way, we may write

$$\mathrm{vol}(\mathbf{X}_5) = \frac{q}{3!}\,\eta \wedge \omega^2 \tag{2.5}$$

where $\eta = d\psi + \frac{3}{q}\sigma$, and $\omega \wedge \omega = \omega^2$.

We want to identify the gauge field A associated with the R-charge. The R-symmetry gauge group is at least as big as U(1), and hence it is a natural guess to associate the U(1) fiber of the Sasaki-Einstein manifold $\mathbf{X}_5$ if not with the R-symmetry group itself, then at least with a subgroup of it. We begin with the inspired guess that redefinitions of ψ be identified with gauge transformations of the R-symmetry gauge field A:

$$\psi \to \psi + \alpha\epsilon \tag{2.6}$$

where

$$A \to A + d\epsilon\,. \tag{2.7}$$

One troublesome issue is that α in general may not be one. To figure out α and to anchor the analysis, we need a geometric object with a definite R-charge. The holomorphic n-form Ω on the noncompact Calabi-Yau cone $\mathbf{Y}_n$ is one such object. Because of its association with the superpotential, Ω has R-charge 2.

The functional dependence of Ω on ψ is very simple: $\Omega \sim \exp(iq\psi)$. A proof will follow in section 2.3. Because Ω has R-charge two, we can directly identify the angular variable ψ_R with a U(1) R-symmetry where now

$$\psi_R \to \psi_R + \epsilon \tag{2.8}$$

and $2\psi_R = q\psi$. Thus $\alpha = 2/q$.

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Including the gauge field F in the supergravity solution means altering the metric

$$ds^2_{\mathbf{X}_5} = \left(\frac{q}{3}\right)^2 \left(d\psi + \frac{3}{q}\sigma + \frac{2}{q}A\right)^2 + h_{\alpha\bar{\beta}} dz^\alpha d\bar{z}^\beta \,. \tag{2.9}$$

The F_5 form flux is also altered

$$\mathcal{F} = 4L^4 \frac{q}{3!}\left(\left(\eta + \frac{2}{q}A\right) \wedge \omega^2 - \frac{1}{3}\, dA \wedge \eta \wedge \omega\right). \tag{2.10}$$

From this expression, we can calculate C_4 to first order in A:

$$C_4 = 4L^4 \frac{q}{3!}\left(\psi\,\omega^2 - \frac{1}{3}\, A \wedge \eta \wedge \omega\right) - \frac{r^4}{L^4} d^4x + \frac{q}{9} L^3 (\star_5 dA) \wedge \eta \,. \tag{2.11}$$

We have introduced some notation: $d^4x = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$, and $\star_5$ is the Hodge star in the AdS_5 directions. This form of F_5 satisfies the SUGRA equations of motion to linear order in A. In particular, F_5 has been constructed such that $d\mathcal{F} = 0$. Notice that $\star\mathcal{F}$ contains a piece proportional to $(\star_5 dA) \wedge \omega$. Hence $d \star \mathcal{F}$ will vanish to linear order in A provided $d \star_5 dA = 0$, i.e. the field equations for the gauge field $F = dA$ are satisfied.

We also may consider the trace of the Einstein equations. The modification of the metric (2.9) causes the Ricci scalar for the full ten dimensional metric to become

$$R \to R + \frac{L^2}{9}|F|^2 \,. \tag{2.12}$$

Thus, the trace of Einstein's equations will contain an extra $|F|^2$ term. This term causes the backreaction of the $\mathrm{U}(1)_R$ field strength on the metric. We ignore it in this paper since it is a second order effect.

As a test of our proposal for the U(1) R-symmetry gauge field, let us calculate the R-charge of a hypothetical baryon in this geometry. We identify the baryons as D3-branes wrapped on 3-cycles inside $\mathbf{X}_5$. In particular, the D3-brane will completely wrap the U(1) fiber of the Einstein-Sasaki manifold $\mathbf{X}_5$ and will also wrap a holomorphic curve in the Kähler-Einstein base V. The dimension of these baryons is proportional to the volume of the 3-cycle C times the curvature length L:

$$\Delta = \mu_3 L^4 \int_C \frac{q}{3}\eta \wedge \omega \,. \tag{2.13}$$

One might expect that the volume is more closely related to the mass of the baryon than to the dimension.[2] We will see later in section 2.4 that the mass receives a correction and that (2.13) is the correct expression.

Now these baryons are thought to be chiral primary operators. Their R-charge should be a multiplicative constant times the dimension. Recall that in a classical lagrangian for a

[2]It is often possible to evaluate (2.13). The integral factors into two pieces: the length of the U(1) fiber and an integral over a holomorphic curve. The length of the U(1) fiber is trivial, 2π. Using the Einstein condition, ω can be related to the first Chern class of V, $c_1(V)$. The second integral becomes an intersection number calculation.

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particle traveling in an electromagnetic field, there is a term of the form $qA_\mu v^\mu$ where q is the charge of the particle and v^μ is its four-velocity. In the lagrangian for a probe baryon of this type, there is a Wess-Zumino term of the form

$$\mu_3 \int_C C_4 = \Delta \frac{2}{3} A \,, \tag{2.14}$$

where we have used (2.13) for Δ.

In the case of $AdS_5 \times \mathbf{X}_5$, we know independently from the supersymmetry algebra that the dimension times 2/3 is the R-charge. The $2\Delta/3$ in the Wess-Zumino term is exactly where the charge should be in a classical lagrangian describing the D3-brane dynamics.

2.2 The R-charge for $AdS_4 \times \mathbf{X}_7$

We now repeat essentially the same analysis for a stack of M2-branes sitting at the tip of a noncompact four complex dimensional cone, $\mathbf{Y}_4$. In general, the eleven dimensional metric is

$$ds^2 = \frac{r^2}{L^2}\eta_{\alpha\beta}dx^\alpha dx^\beta + L^2\frac{dr^2}{r^2} + 4L^2 ds^2_{\mathbf{X}_7} \,, \tag{2.15}$$

where $\eta_{\alpha\beta} = (-++)$ is a Minkowski tensor. Note the additional factors of two scaling the Einstein metric. These factors are necessary to guarantee that the Ricci scalar of the eleven dimensional metric vanishes and will be crucial later on in fixing the R-charge of the baryons.

A solution to the SUGRA equations of motion can be obtained by threading the $\mathbf{X}_7$ space with N units of F_7 form flux. In particular

$$F_7 = 6(2L)^6 \mathrm{vol}(\mathbf{X}_7) \tag{2.16}$$

where $L^6 \sim N$.

We specialize to the case where $\mathbf{X}_7$ is a quasi-regular Einstein-Sasaki space:

$$ds^2_{\mathbf{X}_7} = \left(\frac{q}{4}\right)^2 \left(d\psi + \frac{4}{q}\sigma\right)^2 + h_{\alpha\bar{\beta}}dz^\alpha d\bar{z}^\beta \,. \tag{2.17}$$

We keep the same definition of q and σ. Namely, $d\sigma = 2\omega$ and the number q is defined such that $n\, d\sigma = 2\pi q c_1$ and $n = 4$. With these definitions, $0 \le \psi < 2\pi$. Also, in this way, we may write

$$\mathrm{vol}(\mathbf{X}_7) = \frac{q}{4!}\,\eta \wedge \omega^3 \tag{2.18}$$

where $\eta = d\psi + \frac{4}{q}\sigma$.

Just as in the previous section, redefinitions of ψ can be identified with gauge transformations of the R-symmetry gauge field A:

$$\psi \to \psi + \frac{2}{q}\epsilon \tag{2.19}$$

where

$$A \to A + d\epsilon \,. \tag{2.20}$$

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Including the gauge field F in the supergravity solution changes the metric

$$ds^2_{\mathbf{X}_7} = \left(\frac{q}{4}\right)^2 \left(d\psi + \frac{4}{q}\sigma + \frac{2}{q}A\right)^2 + h_{\alpha\bar\beta}dz^\alpha d\bar z^\beta \,. \tag{2.21}$$

The C_6 form potential becomes

$$C_6 = 6(2L)^6 \frac{q}{4!}\left(\psi\,\omega^3 - \frac{1}{4}\,A\wedge\eta\wedge\omega^2\right) . \tag{2.22}$$

Hence

$$F_7 = 6(2L)^6 \frac{q}{4!}\left((\eta + \frac{2}{q}A)\wedge\omega^3 - \frac{1}{4}\,dA\wedge\eta\wedge\omega^2\right) . \tag{2.23}$$

This form of F_7 satisfies the SUGRA equations of motion to linear order in A. The relevant equations for F_7 are $dF_7 = \frac{1}{2}(\star F_7)\wedge(\star F_7)$ and $d\star F_7 = 0$. We have constructed F_7 such that $dF_7 = 0$. Moreover, $(\star F_7)\wedge(\star F_7)$ and $d\star F_7$ vanish to order A^2 provided the equations of motion for the gauge field $F = dA$ are satisfied, i.e. $d\star_4 dA = 0$. The modification of the metric (2.21) causes the Ricci scalar to become

$$R \to R + \frac{L^2}{4}|F|^2 \,. \tag{2.24}$$

From the standpoint of an effective action in AdS_4, one can see from this modification another way of deriving the equations of motion for the gauge field, $d\star_4 dA = 0$.

To calculate the R-charge of a hypothetical baryon in this geometry, we identify the baryons as M5-branes wrapped on 5-cycles inside $\mathbf{X}_7$. The 5-cycle consists of the U(1) fiber and a holomorphic surface inside the Kähler-Einstein base of $\mathbf{X}_7$. The dimension of these baryons is proportional to the volume of the 5-cycle C times the curvature length L:

$$\Delta = \mu_5(2L)^5 L\int_C \frac{q}{8}\eta\wedge\omega^2 \,. \tag{2.25}$$

In the lagrangian for a probe baryon of this type, there is also a Wess-Zumino term of the form

$$\mu_5\int_C C_6 = \Delta A \,, \tag{2.26}$$

where we have used the previous expression for Δ.

In the case of $AdS_4\times\mathbf{X}_7$, we expect from the supersymmetry algebra that the R-charge of a chiral primary operator be equal to the dimension. As these baryons are chiral primary operators, all is well.

2.3 Proof

We seek to prove that the holomorphic n-form Ω scales as $\exp iq\psi$ where ψ is a coordinate on the U(1) fiber of these non-compact Calabi-Yau cones of complex dimension n. The metric on $\mathbf{Y}_n$ (2.1) is not very natural as it puts r and ψ on an unequal footing. It is more natural to think of $\mathbf{Y}_n$ as a fibration of $\mathbb{C}^*$ over the Kähler-Einstein base V. Let $\lambda\in\mathbb{C}^*$ be the coordinate of the fiber. We make the change of variables

$$\lambda = r^{n/q}e^{i\psi} \,. \tag{2.27}$$

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In these coordinates, the metric becomes

$$ds^2_{\mathbf{Y}_n} = \left(\frac{q}{n}\right)^2 (\lambda\bar\lambda)^{q/n-1}(d\lambda + i\lambda\sigma)(d\bar\lambda - i\bar\lambda\sigma) + (\lambda\bar\lambda)^{q/n} h_{\alpha\bar\beta} dz^\alpha d\bar z^\beta \,. \tag{2.28}$$

The volume form scales as $r^{2n} = (\lambda\bar\lambda)^q$. Note that the volume form is proportional to $\Omega \wedge \bar\Omega$. Thus, Ω scales as λ^q.

2.4 The difference between mass and dimension

So far we have explicitly shown that the supergravity vector field responsible for the $\mathrm{U}(1)_R$ charge of a state results from mixing between the rotations along the S^1 fiber and a compensating gauge transformation. The end result is that the D-brane R-charge was identified with the volume of the D-brane for certain BPS D-branes.

In the supergravity, it seems natural to associate to such a D-brane wrapping the given cycle a mass equal to the volume, and then one finds a discrepancy between the R-charge and the conformal dimension, since

$$\Delta = \frac{d}{2} + \sqrt{m^2L^2 + \frac{d^2}{4}} \tag{2.29}$$

for AdS_{d+1}. We argue that the volume of the D-brane should be identified directly with the conformal dimension, and not with the mass of the particle.

One reason for this is that the D-brane is a BPS object and we should consider the action of a superparticle in this background, rather than the action of a point particle in AdS space. When we consider global AdS coordinates, the hamiltonian associated to time corresponds to the generator of conformal tranformations, and the spectrum is discrete because there is a gravitational potential well at the center of AdS which eliminates the zero modes of the particle and turns them into oscillators.

The supersymmetries do not commute with the hamiltonian, but they still control the dynamics of the theory. This can be seen explicitly in the matrix model presented in [15], where the fermions and bosons are not degenerate, but the ground state energy still cancels. We argue that this is generic for these systems. A superparticle also has fermionic coordinates. It is the zero point energy of these degrees of freedom that cancels the zero point energy of the bosonic degrees of freedom.

In particular, consider the fluctuations corresponding to the transverse motion of the particle in AdS space. The fluctuations have a mass term determined by the AdS_{d+1} metric:

$$ds^2 = -dt^2(\cosh^2\rho) + d\rho^2 + \sinh^2\rho\, d\Omega^2_{d-1} \,. \tag{2.30}$$

If we consider a particle at $\rho = 0$, we have a timelike geodesic of AdS_{d+1}. To quadratic order in the transverse fluctuations we have

$$ds^2 = -dt^2(1 + \vec\rho^{\,2}) + d\vec\rho^{\,2} \,. \tag{2.31}$$

There is a gravitational potential in g_{00}, and the mass term for the fluctuations in $\vec\rho$ is determined by the form of the metric. One can also determine this mass term from

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the algebra of isometries. In the radial quantization, acting with derivatives changes the conformal weight by one unit. This is the same as acting with one of the raising operators for $\vec{\rho}$ to first order in the quantum system, because the symmetry is non-linearly realized.

Similarly, if we act with the supersymmetry generators, we change the conformal weight by a factor of 1/2. This means that the fermions will have a mass term which is one half that of the bosons. There are d complex supersymmetries, so we have one complex fermion associated to each one of them. It is easy to see that the contribution to the conformal weight of each fermion is one half that of the bosons, but there are twice as many fermions. Thus one finds that the zero point energy cancels between bosons and fermions, and for supersymmetric states one has

$$\Delta = \Delta_{\text{class}} = \text{Vol}\,. \tag{2.32}$$

The classical conformal weight associated to the state is equal to the value in the quantum theory, namely, the conformal weight of the superparticle is the volume of the D-brane.

If we integrate out the fermions, they shift the zero point energy of the hamiltonian by $-\frac{1}{2}N_f m_f = -d/2$, where $N_f = 2d$ is the number of real fermions, $m_f = 1/2$ is the mass of each of these fermions, and the remaining $-1/2$ is the numerical factor associated with zero point energy for a fermionic oscillator. As a result, the mass of the particle associated to the ground states is given approximately by

$$mL = \Delta - \frac{d}{2}\,. \tag{2.33}$$

This expression agrees with (2.29) in the limit of large m exactly as we would expect. To first order, integrating out the fermions reproduces the shift between the value of the mass and the conformal weight.

3. Moduli space of D-branes and zero-mode quantization

Classically one often has the freedom to change continuously the way in which a D-brane is wrapped without changing the energy. In this section, we will investigate how, through quantization, this freedom of movement allows one to understand the number of ground states of a given dibaryon operator in the field theory side of the correspondence.

This abstract statement concerning the freedom some dibaryon-like states have to shift their wrapping configurations is particularly easy to understand in the case of $AdS_5 \times T^{1,1}$. Recall that $T^{1,1}$ is a U(1) fibration over $S^2 \times S^2$. The simplest dibaryon corresponds to a D3-brane wrapping the U(1) fiber of one of the two S^2's. It is clear that, classically, the D3-brane can move continuously along the other S^2 without changing its energy.

Let us exhibit the operators of the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory dual to these simply wrapped D3-branes. Since the fields $A^{\alpha}_{k\beta}$, $k = 1, 2$, carry an index α in the $\mathbf{N}$ of $\mathrm{SU}(N)_1$ and an index β in the $\overline{\mathbf{N}}$ of $\mathrm{SU}(N)_2$, we can construct color-singlet "dibaryon" operators by antisymmetrizing completely with respect to both groups:

$$\mathcal{B}_{1l} = \epsilon_{\alpha_1 \ldots \alpha_N} \epsilon^{\beta_1 \ldots \beta_N} D_l^{k_1 \ldots k_N} \prod_{i=1}^{N} A^{\alpha_i}_{k_i \beta_i}\,, \tag{3.1}$$

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where $D_l^{k_1\dots k_N}$ is the completely symmetric SU(2) Clebsch-Gordon coefficient corresponding to forming the $\mathbf{N+1}$ of SU(2) out of N $\mathbf{2}$'s. Thus the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ quantum numbers of $\mathcal{B}_{1l}$ are $(\mathbf{N+1},\mathbf{1})$. Similarly, we can construct "dibaryon" operators which transform as $(\mathbf{1},\mathbf{N+1})$,

$$\mathcal{B}_{2l} = \epsilon^{\alpha_1\dots\alpha_N}\epsilon_{\beta_1\dots\beta_N} D_l^{k_1\dots k_N} \prod_{i=1}^{N} B_{k_i\alpha_i}^{\beta_i}\,. \tag{3.2}$$

The existence of two types of "dibaryon" operators is related on the supergravity side to the fact that the base of the U(1) bundle is $S^2\times S^2$. A D3-brane can wrap either of the two-spheres together with the U(1) fiber.

We will use a simplified notation for the dibaryon operators (3.1)

$$\epsilon^1\epsilon_2(A_{i_1},\dots,A_{i_N})\,. \tag{3.3}$$

The (super)subindex on the ϵ refers to the gauge group that the ϵ symbol carries and whether these are (super)subindices is indicated by the position of the group label. The gauge indices of the A_i are contracted with the gauge indices of the ϵ tensors in the same order as they are written.

In order to account for the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ quantum numbers of the dibaryons, one needs to take into account the proper quantization of the zero modes of the wrapped D3-brane [7]. The resulting dynamics reduces to motion of a particle on a sphere in the presence of a magnetic field. The purpose of this section is to consider the zero mode problem in a more general setting. After discussing the conifold case, we will give a more general argument for other Calabi-Yau singularities that will show that one can count the ground states even without exact knowledge of the metric.

First, we describe this freedom of movement some dibaryons possess in a more precise and useful way. If we begin with dibaryon-like states on $AdS\times\mathbf{X}$, then at each time t, the D-brane worldvolume will be wrapping a holomorphic cycle of $\mathbf{X}/\,\mathrm{U}(1)$, and the volume of the D-brane will be constant, but its shape will change. Since the volume remains constant, there is no potential on these directions, and the moduli space of the D-brane can be captured by the moduli space of holomorphic submanifolds of $\mathbf{X}/\,\mathrm{U}(1)$ with specific topological quantum numbers, which describe what holomorphic cycle is being wrapped.

The D-brane is a holomorphic submanifold of codimension 1 on $\mathbf{X}/\,\mathrm{U}(1)$, so it can be associated to a divisor $[D]$ on $\mathbf{X}/\,\mathrm{U}(1)$. The D-brane is the zero locus of a global section of the associated line bundle to the divisor $[D]$. Any linear combination of global holomorphic sections of $[D]$ is also a global section, and in this way the moduli space of the D-brane is a projective space $\mathbb{CP}^m$ for some m (which can be zero, and then the curve is rigid). m is given by the number of holomorphic sections of the line bundle associated to $[D]$ minus one.

Thus, to quantize the zero modes of the D-brane, we need the effective action of the D-brane moduli space. We consider our action as a nonlinear sigma model whose target space is the moduli space of the D-brane. This action will involve a metric on $\mathbb{CP}^m$ (which generically has some singularities) induced from the metric on $\mathbf{X}/\,\mathrm{U}(1)$. Additionally, one can add a line bundle on this moduli space, that is, an effective magnetic field on the target space of the sigma model.

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In the example of the conifold, for a single dibaryon state $m = 1$, we have the round metric on $\mathbb{CP}^1$ (this is obvious, since we had an S^2 worth of positions for the D-brane with the corresponding isometry group).

Now, we want to argue that the magnetic field through this moduli space is N times the hyperplane bundle on this moduli space. Indeed, the WZ term for the D-brane action is

$$N \int C_{\mu\nu\rho\sigma} dx^\mu dx^\nu dx^\rho dx^\sigma \,. \tag{3.4}$$

Three of the coordinates of the D-brane saturate three of the indices of C_4, let us say $dx^\mu dx^\nu dx^\rho$, and the fourth index is saturated by the time direction in the form

$$\frac{dx^\sigma}{dt} dt \,. \tag{3.5}$$

So the effective action has a term proportional to the velocity on the moduli space of the D-brane. Hence, this WZW term gives rise to an effective magnetic field on the $\mathbb{CP}^m$. Notice moreover that this magnetic field is proportional to N, the effective tension of the D-brane. For the S^2 case of the conifold it is easy to show that the normalization of this magnetic field is such that the flux through the S^2 is N. Counting the number of ground states is then done by the index theorem, and in this case we get $N + 1$. Notice that this matches exactly the counting of operators in the equation 3.3. For the general case we find that the index calculation counts the number of global holomorphic functions of degree N on $\mathbb{CP}^m$. There are

$$\binom{N+m}{N} \tag{3.6}$$

such global sections.

In the case of $\mathbb{C}^3/\mathbb{Z}_3$ orbifold without fixed points [16], we get that $\mathbf{X}/\,\mathrm{U}(1) = \mathbb{CP}^2$. The curves of degree one in $\mathbb{CP}^2$ are parametrized by a dual $\mathbb{CP}^2$. So in this case we get $(N+1)(N+2)/2$ states. These can also be counted in the CFT. Instead of having two fields A_1, A_2 to make the dibaryon, there are three such fields, which transform in the 3 of an SU(3) global symmetry. The totally symmetric combination of N of these objects gives a Hilbert space of the same dimension. Moreover there is an extra degeneracy by a factor of three from the fact that the $S^5/\mathbb{Z}_3$ space is not simply connected, so there is a possibility of having discrete Wilson lines for the gauge field on the D-brane [16]. This factor of three is also reflected in the field theory quiver diagram, where there are three different types of dibaryon states.

Both of these examples show that the counting with N times the hyperplane bundle of $\mathbb{CP}^m$ seems to give the right match for counting states.

With this in mind, let us return to the conifold case, and let us study a D-brane that wraps once both of the two different S^2's. It was argued in [17] that such D-branes should become giant gravitons. The counting of holomorphic sections of the line bundle gives us $m = 3$. Indeed, if we consider x_1, x_2 and y_1, y_2 a set of projective coordinates for both of the S^2, the global sections of this line bundle are given by curves

$$\sum a_{ij} x_i y_j = 0 \tag{3.7}$$

where we get four possible distinct coefficients a_{ij}.

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The number of such states is therefore $(N+1)(N+2)(N+3)/3!$. This is many more states than just the product of two dibaryons, one for A and one for B. Indeed, these factorized states correspond to the curves that factorize into linear terms in the equation 3.7. But when we have intersecting branes, we have also deformations that are localized at the intersection. These are the ones responsible for giving us a larger moduli space.

In the case where the curve factorizes, the operator is given by

$$\epsilon^1 \epsilon_2(A, \ldots, A)\, \epsilon^2 \epsilon_1(B, \ldots, B)\,. \tag{3.8}$$

Notice now that the total baryon number[3] of this operator is zero, and that the ϵ^1 appears as many times as the ϵ_1. We can use the identity

$$\epsilon^{a_1 \ldots a_N} \epsilon_{\tilde{a}_1 \ldots \tilde{a}_N} = \sum_\sigma (-1)^{|\sigma|} \delta^{a_1}_{\sigma(\tilde{a}_1)} \ldots \delta^{a_N}_{\sigma(\tilde{a}_N)} \tag{3.9}$$

where σ sums over all possible permutations of the $\tilde{a}_i$, and $|\sigma|$ is 0 or 1 if the permutation is even or odd.

This procedure lets us eliminate one set of the ϵ symbols if we want to. We see that if we do this, the A have to be paired with the B in combinations of the form AB. There are 4 such possible combinations. Consider therefore the operators of the form

$$\epsilon^1 \epsilon_1((AB), (AB), \ldots, (AB)) \tag{3.10}$$

where each of the entries is given by one of the four possible combinations $A_i B_j$. This object can be expressed in terms of traces of the AB, so it is built out of gravitons, and indeed it should represent a maximal giant graviton. We have a symmetric combination of four objects, and the combinatorics of these objects also gives us a total of $\binom{N+3}{3}$ objects. In some sense, A_i and B_j are the analog of the coordinates x_i and y_j.

Similarly, if we take a state which wraps sphere one twice and sphere two once, we get that $m = 3 \times 2 - 1$. The natural objects to consider are of the form $A_i B_j A_k$. However, we have to remember the F-terms, so in this expression we get a field which is symmetric in i, k. The total number of objects that we can count is 6, and we can build operators of the form

$$\epsilon^1 \epsilon_2((ABA), (ABA), \ldots, (ABA))\,. \tag{3.11}$$

Notice that this operator corresponds to some form of a coherent state of excitations of the dibaryon (3.3). From the supergravity, we expect such a result because this new baryon-like state is wrapping the same topological cycle as the old dibaryon.

For more general wrapping cases of baryon number two or higher, it is more difficult to match the states. Since we cannot get rid of all but one of the pairs of ϵ symbols, it is not true that there is an easy description of the state in terms of N-symmetrized products of similar objects (the ABA above). It would be interesting to obtain a more thorough description of these other states.

[3]The baryon number counts the number of A-fields minus the number of B fields, with a choice of normalization which we set equal to one for the smallest dibaryon operator.

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4. BPS fluctuations of dibaryons

In this section we show that there exist BPS excitations of the dibaryon operators, that is, operators which carry baryon number and have higher conformal weight than the volume of the associated 3-cycle. Since in this particular section we will be interested in computing explicitly the fluctuation spectra of D-branes, we need to resort to writing models for specific examples where the metric and the CFT are both known. We are moreover interested in situations where we have only $\mathcal{N} = 1$ supersymmetry without singularities. In these models the R symmetry is strictly U(1) and the chiral fields are holomorphic. Two of these models are particularly simple. These are D-branes at the conifold [6] and D-branes at the orbifold $\mathbb{C}^3/\mathbb{Z}_3$ [16] without fixed points. We will deal in this section with the particular case of the conifold.

Let us consider for simplicity the state with maximum J_3 of the first SU(2):

$$\epsilon^1\epsilon_2(A_1, \ldots, A_1) = \det A_1\,. \tag{4.1}$$

To construct excited dibaryons we can replace one of the A_1 by any other chiral field which transforms in the same representation of the gauge groups. One possibility is to replace $A_1 \to A_1B_iA_j$, where the two gauge indices of B are contracted separately with the A_1 and with the A_j, following the rules for matrix multiplication, namely

$$(A_1B_iA_j)^{b_1}_{a_2} = (A_1)^{b_1}_{a_1}(B_i)^{a_1}_{b_2}(A_j)^{b_2}_{a_2}\,. \tag{4.2}$$

This dibaryon-like state is a chiral field up to F-terms because it is a gauge invariant holomorphic polynomial in the chiral superfields. There will be a chiral primary state with the same quantum numbers as the above operator that is a linear combination of operators of this type. Remember that operators that differ by F-terms are equivalent as elements of the chiral ring, but in the conformal field theory they are different and only one particular linear combination is protected.

The operator resulting after the replacement factorizes into the original dibaryon and a single-trace operator:

$$\mathrm{Tr}(B_iA_j)\ \det A_1\,. \tag{4.3}$$

The factorization suggests that this excitation of a dibaryon can be represented as a graviton fluctuation in presence of the original dibaryon.

Not all excitations factorize in this way, however. For example, consider replacing $A_1 \to A_2B_iA_2$. One might ask whether this new operator can be written as a product of the original dibaryon $\det A_1$ and a meson-like operator of the form $\mathrm{tr}(A_2B_i)$. The answer is no. To see why, it is easier to go to a generic point in the moduli space of vacua of the theory where we can set $A_1 = \mathbf{1}a$ by gauge transformations, and thus we establish an isomorphism between the indices of the two gauge groups. The operator above becomes $a^{n-1}\,\mathrm{tr}(A_2B_iA_2)$, and if N is large enough, there are no relations amongst traces of a low number of fields. In other words, we cannot write the operator as $\epsilon^1\epsilon_2(A_1, \ldots, A_1, A_2)\,\mathrm{tr}(B_iA_2) \sim a^{n-1}\,\mathrm{tr}(A_2)\,\mathrm{tr}(B_iA_2)$. Since this new operator cannot be

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factored it has to be interpreted as a single particle state in AdS.[4] Since the operator also carries baryon number one, the natural conclusion is that the one-particle state is a BPS excitation of the wrapped D3-brane in the dual string theory.

Let us now study BPS fluctuations of the wrapped D3-brane in the supergravity approximation. The DBI action is a good approximation in the limit of weak string coupling and weak curvature of the D-brane. These conditions are met in the limit of large 't Hooft coupling. In particular, we will compute the spectrum of quadratic fluctuations of this DBI action. We will return to the field theory later to make the correspondence of states and quantum numbers more precise.

First, we set up the DBI computation in the right coordinate system. The full ten dimensional metric is naturally $AdS_5 \times T^{1,1}$,

$$ds^2 = L^2 \left(-\cosh^2(\rho)d\tau^2 + d\rho^2 + \sinh^2(\rho)d\Omega_3^2\right) + L^2 g\,. \tag{4.5}$$

For convenience, we have chosen the time τ direction to be a Killing vector. The radius of curvature is L. The metric of $T^{1,1}$, the base of the conifold, is given by

$$g = b^2\left[A^2\left(d\psi + \cos(\theta_1)d\phi_1 + \cos(\theta_2)d\phi_2\right)^2 + \sum_{i=1}^{2}\left[d\theta_i^2 + \sin^2(\theta_i)d\phi_i^2\right]\right] \tag{4.6}$$

with $A^2 = 2/3$, $b^2 = 1/6$. We will keep A, b as variables in our computation for consistency checks.

The dibaryon is chosen to wrap the cycle defined by θ_2, ϕ_2 constant. This configuration is invariant under rotations of the sphere wrapped by the D3-brane, but it is not invariant under the SU(2) associated to the θ_2, ϕ_2 coordinates. The induced metric on the dibaryon is thus

$$(Lb)^{-2}g_{\rm ind} = -b^{-2}\cosh^2(\rho)d\tau^2 + A^2(d\psi + \cos(\theta_1)d\phi_1)^2 + (d\theta_1^2 + \sin^2(\theta_1)d\phi_1^2)\,. \tag{4.7}$$

It is convenient to make a change of variables $\cos(\theta_1) = x$, so that

$$(Lb)^{-2}g_{\rm ind} = -b^{-2}\cosh^2(\rho)d\tau^2 + A^2(d\psi + xd\phi_1)^2 + \left[\frac{dx^2}{1-x^2} + (1-x^2)d\phi_1^2\right]. \tag{4.8}$$

In these variables, the determinant of the spatial part of the metric

$$\frac{-\det g_{\rm ind}}{L^2\cosh^2(\rho)} = (Lb)^6 A^2 \tag{4.9}$$

is constant. The variables' range is given by $\psi \in [0, 4\pi)$, $\phi_1 \in [0, 2\pi)$ and $x \in [-1, 1]$. The volume of the wrapped manifold is thus $L^3 16\pi^2 A b^3 = 8L^3\pi^2/9$. In [7], it was noted

[4]Another way of seeing that this new operator is not a multiparticle state begins with the observation that in general

$$\epsilon^1\epsilon_2(A_1, A_1, \ldots, A_1, C) = \frac{1}{N}\,\mathrm{tr}(A_1^{-1}C)\epsilon^1\epsilon_2(A_1, A_1, \ldots, A_1) \tag{4.4}$$

where C is some operator with the same transformation properties as the A_i. From this formula, it is clear that if $C = A_1B_iA_1$, $A_1B_iA_2$, or $A_2B_iA_1$, the new operator factors inside the ring of chiral primaries. However, for the case we picked above, $C = A_2B_iA_2$, the A_1^{-1} cannot be eliminated and the operator does not factor.

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that this volume times the tension of the D3-brane should be a good approximation of the mass of the corresponding dibaryon. As we argued in section 2.4, this volume is directly proportional to the dimension Δ of the dibaryonic operator. Indeed, as the conformal dimension of each A_i and B_i is 3/4, one finds that $\mu_3 8L^4\pi^2/9 = 3N/4$ is exactly equal to Δ. We return now to calculating the excitation spectrum of the dibaryon.

We need to be careful with single valued functions on this space. This squashed S^3 can be thought of, essentially, as the group manifold SU(2). The coordinates (ψ, θ_1, ϕ_1) are the Euler angles. One might have thought that the points (ψ, θ_1, ϕ_1) and $(\psi, \theta_1, \phi_1 + 2\pi)$ were equivalent. The insight from SU(2) lets us correct this mistake. Equivalent points on SU(2) have

$$\psi + \phi_1 \equiv \psi' + \phi_1' \mod 4\pi\,, \tag{4.10}$$

$$\psi - \phi_1 \equiv \psi' - \phi_1' \mod 4\pi\,, \tag{4.11}$$

where (ψ, θ, ϕ) and (ψ', θ', ϕ') are two points on SU(2).

We want to find the normal modes of oscillation of the wrapped D3-brane around the solution corresponding to some fixed world-line in AdS_5 and some fixed θ_2 and ϕ_2 on the transverse S^2. The fluctuations along the transverse S^2 are the most interesting: they change the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ quantum numbers and are most usefully compared with the chiral primary states in the field theory. The transverse fluctuations along the AdS_5 are considered briefly afterward. Supersymmetry relates the gauge field degrees of freedom and fermions on the D-brane to the scalar modes considered here. There is no mixing between the between the different modes at quadratic order in the fluctuations, and we ignore the vector and spinor modes in what follows.

Because the fluctuations around ϕ_2 and θ_2 are by definition small, it is appropriate to treat the S^2 parametrized by these coordinates as a flat $\mathbb{R}^2$. For example, one may take $\theta_2 \approx 0$. Then the fluctuation coordinates on S^2 can be taken to be $y_1 = \theta_2 \sin(\phi_2)$ and $y_2 = \theta_2 \cos(\phi_2)$. The connection term in the metric on $T^{1,1}$ becomes $\cos(\theta_2) d\phi_2 = d\phi_2 + \frac{1}{2}(y_1 dy_2 - y_2 dy_1)$ and the Kähler form on the transverse S^2 becomes $\sin(\theta_2) d\phi_2 \wedge d\theta_2 = dy_1 \wedge dy_2$. To say the same thing in a different way, we are interested in the D-brane action to quadratic order in fluctuations. We only need to consider terms of up to order y^2, ydy, or dy^2, and we neglect everything else which is higher order in the fluctuations y.

Notice that the ten dimensional metric (4.5) has a term of the form $(d\psi + A_\mu dx^\mu)^2$, where the x^μ are the ϕ_1 and ϕ_2 coordinates. Indeed, in the previous section, we saw that we could add a more general term of this form depending on the AdS_5 coordinates. This perturbation corresponded to a gauge field carrying the R-charge. Recall that changing the coordinate ψ to $\psi' = \psi + f(x)$ does not change the periodicity of the variable ψ'; but it does change the form of the vector A_μ by a gauge transformation, $A_\mu \to A_\mu - \partial_\mu f$. So in writing the metric, the invariant quantity is the field strength of A. We are in this way free to add a term of the form $-d\phi_2$ to A, changing $d(\cos(\theta_2) d\phi_2) = d(\frac{1}{2}(y_1 dy_2 - y_2 dy_1))$ and making the connection term simpler.

The Dirac-Born-Infeld acion is given by

$$S_{\text{eff}} = -\mu_3 \int d^4x\, \sqrt{-\det g_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu} + \mu_3 \int C_4 \tag{4.12}$$

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with μ_3 the tension of the brane. Because y_1, y_2 appear only quadratically in the metric, we can study the quadratic fluctuations in y by doing a first order variation in the metric

$$S_{\text{eff}} = S_0 - \frac{\mu_3}{2} \int d^4x \sqrt{-\det g^{(0)}_{\text{ind}}}\ \text{tr}\left[g^{-1}_{ind} \delta g_{\text{ind}}\right] + \mu_3 \int C_4 \tag{4.13}$$

where δg_{ind} is the second order contribution from the fluctuations in y_1, y_2 and we define g^{-1}_{ind} to be the inverse of $g^{(0)}_{\text{ind}}$. We can also do the same with the transverse fluctuations along the AdS_5, but we will leave those for later.

With the coordinates chosen $\sqrt{-\det g^{(0)}_{\text{ind}}}$ is independent of the fluctuations y_1, y_2 and also independent of the D3-brane coordinates. As a result, the wave equation on the D-brane worldvolume is simplified. We fix the diffeomorphism invariance of the DBI action by locking the internal brane coordinates to the background coordinates ψ, ϕ_1, x, τ. This locking corresponds to choosing a physical gauge. We take $\rho = 0$. If the brane is not moving then

$$L^{-2} g^{(0)}_{\text{ind}} = \begin{pmatrix} A^2b^2 & A^2b^2x & 0 & 0 \\ A^2b^2x & b^2(1-x^2) + A^2b^2x^2 & 0 & 0 \\ 0 & 0 & \frac{b^2}{1-x^2} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{4.14}$$

The inverse matrix is

$$L^2 g^{-1}_{\text{ind}} = \begin{pmatrix} \frac{1-x^2+A^2x^2}{A^2b^2(1-x^2)} & -\frac{x}{b^2(1-x^2)} & 0 & 0 \\ -\frac{x}{b^2(1-x^2)} & \frac{1}{b^2(1-x^2)} & 0 & 0 \\ 0 & 0 & \frac{1-x^2}{b^2} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{4.15}$$

We may choose a gauge where C_4 has a piece in the ψ, θ_1, ϕ_1 and ϕ_2 directions. More specifically, we will choose a gauge which is well defined at the north pole of the sphere parametrized by θ_2 and ϕ_2:

$$C_4 = 4Ab^5L^4\left(-1 + \cos(\theta_2)\right)\sin(\theta_1) d\psi \wedge d\theta_1 \wedge d\phi_1 \wedge d\phi_2 \,. \tag{4.16}$$

We want to expand the probe brane action to quadratic order in fluctuations in y_1 and y_2. At this order, the four form C_4 is approximately

$$C_4 \approx -2b^2 \sqrt{-\det g^{(0)}_{\text{ind}}}\, (y_1 dy_2 - y_2 dy_1) d\psi \wedge dx \wedge d\phi_1 \,. \tag{4.17}$$

Putting the various pieces together, we obtain that

$$\text{tr}\, g^{-1}_{\text{ind}} \delta g \;= \sum_i g^{\alpha\beta}_{\text{ind}} L^2 b^2 (\partial_\alpha y_i \partial_\beta y_i) + 2 g^{\alpha\beta}_{\text{ind}} g_{\alpha i} \partial_\beta y^i \,. \tag{4.18}$$

The first term gives rise to the standard laplacian on the three-sphere, while the second term gives rise to mixing terms between y^1, y^2 that are only first order in derivatives. An additional mixing term comes from (4.17). These three contributions give us the effective lagrangian density to quadratic order in fluctuations.

$$\mathcal{L} = \text{tr}\, g^{-1}_{\text{ind}} \delta g + 4b^2 (y_1 \partial_\tau y_2 - y_2 \partial_\tau y_1) \,. \tag{4.19}$$

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The non-trivial elements of $g_{\alpha i}$ are

$$g_{\psi i} = -\frac{1}{2} L^2 A^2 b^2 \epsilon_{ij} y^j \, ; \qquad g_{\phi_1 i} = -\frac{1}{2} L^2 A^2 b^2 x \epsilon_{ij} y^j \, . \tag{4.20}$$

From now on, we will call ϕ_1 simply ϕ. The spatial mixing term $2 g_{\rm ind}^{\alpha\beta} g_{\alpha i} \partial_\beta y^i$ is explicitly given by

$$-\frac{L^2}{2} \Big[g^{\psi\psi} A^2 b^2 y^2 \partial_\psi y^1 + g^{\psi\phi} A^2 b^2 y^2 \partial_\phi y^1 + g^{\phi\psi} A^2 b^2 x y^2 \partial_\psi y^1 + g^{\phi\phi} A^2 b^2 x y^2 \partial_\phi y^1 - (1 \leftrightarrow 2) \Big] . \tag{4.21}$$

The equations of motion for the fluctuations are given by

$$L^2 b^2 \partial_\alpha (g_{\rm ind}^{\alpha\beta} \partial_\beta y^i) + \partial_\beta (g_{\rm ind}^{\alpha\beta} g_{\alpha i}) - (\partial_{y_i} g_{\alpha i}) g_{\rm ind}^{\alpha\beta} \partial_\beta y^i - 4 b^2 \epsilon^{ij} \partial_\tau y^j = 0 \, . \tag{4.22}$$

Notice that the elements of $g_{\rm ind}^{\alpha\beta}$ are independent of the variables with respect to which we are taking derivatives. As a result, the second and third terms in the equation above are actually equal and we have

$$L^2 b^2 \partial_\alpha (g_{\rm ind}^{\alpha\beta} \partial_\beta y^i) + 2 \partial_\beta (g_{\rm ind}^{\alpha\beta} g_{\alpha i}) - 4 b^2 \epsilon^{ij} \partial_\tau y^j = 0 \, . \tag{4.23}$$

Taking the combinations $y^\pm = y^1 \pm i y^2$ these equations become

$$\begin{aligned} b^2 (\nabla^2 y^\pm - L^{-2} \partial_\tau^2 y^\pm) \mp i 4 b^2 L^{-2} \partial_\tau y^\pm \pm \\ \pm i b^2 A^2 \left(g^{\psi\psi} \partial_\psi y^\pm + g^{\psi\phi} x \partial_\psi y^\pm + g^{\psi\phi} \partial_\phi y^\pm + g^{\phi\phi} x \partial_\phi y^\pm \right) = 0 \, . \end{aligned} \tag{4.24}$$

Now, we see that $g^{\psi\phi} = -x g^{\phi\phi}$ so the terms with derivatives with respect to ϕ cancel. This is just as expected, since the result should be invariant under the SU(2) of isometries of the squashed S^3. Thus far we are getting a consistent picture.

The terms with $\partial_\psi y^\pm$ are given by the combination

$$g^{\psi\psi} + g^{\psi\phi} x = \frac{1}{L^2 A^2 b^2} \tag{4.25}$$

which is a constant coefficient.

Now we can use the separation of variables

$$y^\pm = \exp(-i\omega\tau) \exp(i m \psi) \exp(i n \phi) Y_{mn}^{k\pm}(x) \tag{4.26}$$

to obtain a differential equation for $Y_{mn}^{k\pm}(x)$. Note that for the $y^\pm$ to be single valued, the condition (4.11) implies that m and n are either both integer or both half-integer. The remaining differential equation for the $Y_{mn}^{k\pm}$ is given by

$$\begin{aligned} &\partial_x (1 - x^2) \partial_x Y_{mn}^{k\pm} = \\ &\quad = b^2 \left(\frac{1 - x^2 + A^2 x^2}{A^2 b^2 (1 - x^2)} m^2 - \frac{2x}{b^2 (1 - x^2)} m n + \frac{n^2}{b^2 (1 - x^2)} - \omega(\omega \pm 4) \pm b^{-2} m \right) Y_{mn}^{k\pm} \, . \end{aligned} \tag{4.27}$$

Let us begin by analyzing the behavior of the solution in the limit $x \to \infty$. In this limit, the differential equation becomes

$$b^2 \left(\frac{1 - A^2}{(A^2 b^2)} m^2 - \omega(\omega \pm 4) \pm b^{-2} m \right) Y_{mn}^{k\pm} + \partial_x (x^2 \partial_x Y_{mn}^{k\pm}) = 0 \, . \tag{4.28}$$

The solution to this equation is clearly a power of x, $Y^{k\pm}_{mn} \sim a_k x^k$. Thus

$$\omega(\omega \pm 4) = b^{-2}\left(k(k+1) - m(m \mp 1)\right) + \frac{m^2}{A^2 b^2}\,. \tag{4.29}$$

The energy should be real for all allowed values of m and k. Notice that the energy has the right dependence in terms of the SU(2) quantum numbers to be associated with the velocity on a group manifold.

Only certain values of k are allowed because the fluctuations must be well-behaved at $x = \pm 1$. The differential equation (4.27) can be solved in terms of a hypergeometric function

$$Y^{k\pm}_{mn} = (1-x)^{\delta}(1+x)^{\epsilon} F\left(\alpha, \beta, \gamma; z = \frac{(x+1)}{2}\right), \tag{4.30}$$

where α, β, γ, δ, and ϵ depend on the quantum numbers k, m, and n. The north and south pole of the S^2, $x = \pm 1$, correspond to the singular points $z = 0$ and $z = 1$ of the hypergeometric function. For the fluctuations to vanish at $x = \pm 1$, $k - m$ must be a non-negative integer.[5]

For the $y^{\pm}$ which contribute to the U(1) charge in the same direction as the unexcited D3-brane, the choice $k = m$ corresponds to a BPS state; we have

$$\omega = \frac{m}{Ab} = 3m\,. \tag{4.31}$$

Indeed, the BPS states should have the lowest possible dimension for a given R-charge. The states $k = m$ meet this condition. From the periodicity of ψ, m can be a half integer, and when m is a half integer so is k.

This energy spectrum means that the contribution of these BPS states to the energy is quantized in units of $3/2L^{-1}$. $3/2$ is also exactly the change in the conformal dimension of the chiral operators once $A_2B_iA_2$ is substituted for an A_1 in the antisymmetric product (4.1). These modes match the result from conformal field theory.[6]

Also, the transformation under the SU(2) that rotates B_1, B_2 is in agreement. The unexcited dibaryon was a singlet while the excited one acquires spin $k = m$. Indeed, for each value of m there is a unique irreducible representation of $\mathrm{SU}(2)_B$ associated to it. This means that the quantum of U(1) charge proportional to m should be associated to a spin m state for $\mathrm{SU}(2)_B$. In the conformal field theory, these states result from choosing to replace one of the A_1 in the $\epsilon^1\epsilon_2(\ )$ function by, for example, $A_2B_iA_2B_k \ldots A_2$, where we have $2m$ B's inside the matrix. Using the F-term equations of motion, it can be seen that these states are totally symmetric with respect to exchange of the B variables. Since the $B_{1,2}$ carry spin 1/2 under the $\mathrm{SU}(2)_B$, these states with $2m$ B's form a totally symmetric representation of $\mathrm{SU}(2)_B$ with spin m. Therefore, we can match the $\mathrm{SU}(2)_B$ quantum numbers of the states to the supergravity.

[5]The word "must" is a little too strong here. In fact, a certain symmetry in the functional form of (4.30) means that distinct choices of k, m, n may correspond to the same fluctuation. To be more precise, the choice $k - \max(m, n) \in \{0, 1, 2, \ldots\}$ and $m + n > 0$ corresponds to non-singular fluctuations with no redundancy.

[6](AB) contributes $3/2 \times 2/3$ to the R-charge of the state.

We have not, however, determined the transformation property of the excited dibaryons under the SU(2) that rotates A_1, A_2. This is more difficult since we need to consider the coupling of the fluctuation fields with the zero-mode dynamics. Since we have not determined the full $\mathrm{SU}(2) \times \mathrm{SU}(2)$ quantum numbers, we cannot make a precise determination of the dual gauge theory operators. In particular, it would be interesting to see if the factorized operators of the type (4.3) are necessary to match the spectrum of the fluctuating probe D3-brane. We leave this interesting question for the future.

For the transverse motion in AdS_5 there is no mixing of the directions, so we get four scalars and their energies are given by

$$\omega^2 = b^{-2}(k(k+1) - m^2) + \frac{m^2}{A^2 b^2} + M^2 \tag{4.32}$$

with M the mass of these states. This mass term comes because the space-time is curved and to second order $g_{00} = -L^2(1 + \rho^2 + \cdots)$; the AdS excitation feels a gravitational potential. For $k = m$ the above expression should be a perfect square for all m (it is a superpartner of the other BPS states), so we should have $M^2 A^2 b^2 = A^4/4$, or equivalently $M^2 = A^2/4b^2 = 1$. This relation can be checked explicitly from the metric. It follows that $\omega = 3m + 3A^2/2 = 3m + 1$ differs by 1 from the previous energy. Indeed, we expect excitations in the AdS directions to correspond to introducing covariant derivatives for the fields A_i or $A_2 B_i A_2$ on the field theory side. Covariant derivatives have conformal weight one. This is an alternative check that the normalization of the C field we chose is correct, since it predicts that the splitting between the energies of the modes is compatible with the spacetime symmetries.

Notice that the dibaryon state on the gravity side has a Fock space worth of excitations. In the field theory, this Fock space can be reproduced as well. To insert one quantum, we took one of the A_1 and replaced it with $A_2 B A_2 B \ldots$. To put many quanta, we replace many of the A_1 by the $A_2 B A_2 B A_2 \ldots$ combinations. The fact that we have a Fock space of identical particles, as opposed to distinguishable particles, comes from the permutation symmetry of the ϵ symbols. For fermionic excitations we need to remember the $(-)$ signs when we exchange the fermionic insertions in the operator.

We conclude that we can match, at least schematically, the dibaryon state with any number of BPS open strings that excite it. We have not carried out the complete matching since we have not determined the $\mathrm{SU}(2)_A$ quantum numbers of the excited D3-branes, which remains an interesting problem for future work. We believe that the construction of non-BPS states can be carried out using similar methods to the ones used in [15] for closed strings, but this construction is beyond the scope of the present paper.

5. Discussion

In this paper we studied in some detail the correspondence between D3-branes wrapping 3-spheres inside $T^{1,1}$ and baryon-type operators of the dual $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory. By calculating the $\mathrm{U}(1)_R$ charge of the D3-branes and their collective coordinate energies we have new evidence that the operator identification proposed in [7] is correct. Furthermore,

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we showed that there exist BPS excitations of wrapped D3-branes and suggested the chiral operators dual to them. Our results provide new evidence that the duality between gauge invariant operators of a superconformal gauge theory and states of string theory on $AdS_5 \times \mathbf{X}_5$ extends to operators whose dimensions grow as N, the number of colors. It is clear that this same procedure to generate operators should work in other examples related to different quiver theories and that one can generically find BPS excitations of baryon-like operators.

There is another such class of operators, similar to the ones we have considered. It is related to the "giant graviton" effect [18]. It has been observed that for modes whose angular momentum on $\mathbf{X}_5$ is of order N the single-trace description of the dual gauge theory operators breaks down [17, 19]. Instead, the correct description is in terms of subdeterminants of elementary fields, which for maximum angular momentum become determinants similar to the dibaryon operators we have considered. On the string theory side, the modes whose angular momentum is of order N blow up into D3-branes on $\mathbf{X}_5$.[7] This is another manifestation of the fact that D-branes, and therefore string theory, are crucial for describing gauge invariant operators whose dimensions grow as N. Thus, string theory is necessary to complete the state/operator map even at large 't Hooft coupling. Along these lines, it is interesting to consider the BPS excitations of the giant gravitons and to construct the dual operators. Some results on this are presented in [20]. We believe that the giant graviton case is similar to our study of topologically stable wrapped D3-branes.

Very recently, a much more dramatic demonstration of the stringy nature of the AdS/CFT duality at large 't Hooft coupling was presented in [15]. The insight of this paper is to focus on states whose angular momentum J on S^5 scales as $\sqrt{N}$. It was shown that there exists a class of operators whose $\Delta - J$ stays finite in this limit. These states are in one-to-one correspondence with *all* the closed string states on a RR-charged pp-wave background, including all the massive string states. We believe that the work we have presented on the BPS excitations of D3-branes constitutes a first step towards studying the open string states in a similar setting. An unexcited D3-brane is described by the basic dibaryon operators (3.1) and (3.2). Thus, these operators describe the open string vacuum. The more complicated operators discussed in section 4 correspond to the BPS states of the open string. The non-BPS open string states can be constructed along the lines of [15], but this construction is beyond the scope of the present paper. The basic issue in question is the proper understanding of the non-planar diagrams. When the operator dimension scales as N, it is too high for the planar approximation to be valid.

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[7]These objects are not topologically stable. The blow-up happens for dynamical reasons.

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